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On-Mass-Shell Gaugino Condensation in Z_N Orbifold Compactifications

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Abstract

We discuss non-perturbative aspects of string effective field theories with $N = 1$ supersymmetry in four dimensions. By the use of a scalar potential, which is on-shell invariant under the supersymmetric duality of the dilaton, we study gaugino condensation in $(2, 2)$ symmetric Z_N orbifold compactifications. The duality under consideration relates a two-form antisymmetric tensor to a pseudoscalar. We show, that our approach is independent of the superfield-representation of the dilaton and preserves the $U(1)_{PQ}$ Peccei-Quinn symmetry exactly.

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We discuss effective quantum field theories (EQFT's) of strings with local N=1 supersymmetry in four dimensions. These theories are effective in the sense, that they are low-energy limits of a given higher dimensional string theory after dimensional reduction and integrating out all heavy modes. We restrict ourselves to the case of EQFT's, which are only of second order in derivatives in the bosonic fields. In these EQFT's the tree level gauge coupling constant is dynamical and can be expressed by the vacuum expectation value of the dilaton superfield. The dilaton superfield can be represented by the more familiar chiral superfield S or the linear superfield L: $g_{tree}^2 = 2\,\langle S+\bar{S}\rangle^{-1} = \langle L\rangle$. Throughout this paper $S+\bar{S}$ will be denoted as the chiral representation of the dilaton (S-representation) and L as the linear representation (L-representation).
It has been shown that these two superfield representations of the dilaton are connected via a supersymmetric legendre transformation called *supersymmetric duality* [1, 2, 3, 4, 5, 6]. This duality transformation destroys the holomorphic structure of the tree level gauge coupling constant. We will focus on the supersymmetric duality of the dilaton as an on-shell duality. In the simplest form it relates the two-form antisymmetric tensor $b_{mn}(x)$ to a pseudoscalar, the so called axion $a(x)$, via the following algebraic equation:

$$\partial_m \; a(x) \;\; = \;\; - \, \varepsilon_{mnpq} \; \partial^n \; b^{pq}(x) \tag{1.1}$$

The two-form antisymmetric tensor is a physical degree of freedom and plays an important role in the four dimensional formulation of the Green-Schwarz anomaly cancellation mechanism [4,5].
Since one integrates out only the massive states to go to the effective theory, the universal degrees of freedom, namely the graviton, the antisymmetric tensor and the dilaton, appear in the low energy supergravity action of four-dimensional, $N = 1$ supersymmetric heterotic strings. Due to phenomenological reasons $N = 1$ supersymmetry must be broken and gaugino condensation provides a promising mechanism for spontaneous supersymmetry breaking [7]. At the level of an string effective supergravity action this has been studied extensively in the S-representation of global and local supersymmetry [8,9,10].
Because the L-representation contains a two-form antisymmetric tensor this formulation seems to be the more natural EQFT of strings. It has been shown, that this EQFT most directly reproduces results calculated in the underlying string theory [11, 12]. Thus, it is an important task to formulate supersymmetry breaking by gaugino condensation in the linear representation of the dilaton. This issue was recently discussed by us [13], for some other interesting considerations see also [6, 14].
In this context we have to face a puzzle: Gaugino condensation is known to be a non-perturbative effect producing an effective scalar potential $V \sim e^{-1/g^2}$. This can be achieved in the S-representation at tree level by a non-perturbative superpotential $\omega_{np} \sim e^{-S}$, because S is a chiral superfield. This is - first of all - impossibel in the L-representation, because the linear multiplet is not chiral and can therefore not enter the superpotential. On the other hand the duality (1.1) is independent of the dilaton at component level and contains derivatives. How can then the e^{-1/g_{tree}^2} dependence of the effective scalar potential be spoilt by the duality transformation?

We will show in the following that the formation of gaugino condensates can be consistently formulated in both representations of the dilaton. However, our approach only works on-shell. That is to say, the off-shell structure of the discussion given here is still an open problem in many aspects. One important result of our approach is, that the well-known $U(1)_{PQ}$ Peccei-Quinn symmetry is exactly preserved:

$$\begin{aligned} b_{mn}(x) &\rightarrow b_{mn}(x) + \partial_m b_n(x) - \partial_n b_m(x) && (1.2)\\ a(x) &\rightarrow a(x) + \Theta \qquad\qquad , \quad \Theta \in \mathbf{R} && (1.3)\\ b_{mn}(x) &\rightarrow b_{mn}(x) + c_{mn} \qquad , \quad c_{mn} \in \mathbf{R} && (1.4) \end{aligned}$$

Note that (1.2) is known to be the gauge symmetry of a two-form antisymmetric tensor, whereas (1.3) and (1.4) are global shift symmetries of the axion and the antisymmetric tensor respectively. The on-shell duality transformation (1.1) is invariant under the $U(1)_{PQ}$ symmetry.
The paper is organized as follows: After a short introduction of various $N = 1$ off-shell multiplets we discuss the supersymmetric duality of the dilaton. These results are already well-known. Then we derive a duality-invariant scalar potential and study gaugino condensation in $(2, 2)$ symmetric Z_N orbifold compactifications. We present a duality-invariant discussion of gaugino condensation for several gauge groups[1].

A general supersymmetric multiplet Z has the following structure at component level: $Z \sim$ (Bosons | Fermions || Auxiliary-Fields). One of the basic objects in any superspace formulation [2,15,16] are chiral $4_B + 4_F$ multiplets $\Sigma \sim (A \mid \chi_\alpha \mid\mid F)$, because their lowest components are scalar fields parametrizing a Kähler manifold [17]. These chiral superfields obey the constraint $\bar{\mathcal{D}}^{\dot\alpha}\, \Sigma = 0$ and are defined at component level as

$$\Sigma_| \;=\; A(x) \qquad \mathcal{D}_\alpha \Sigma_| = \sqrt{2}\; \chi_\alpha(x) \qquad \mathcal{D}^2 \Sigma_| = -4\; F(x). \tag{1.5}$$

We will denote in the following all possible chiral superfields by Σ except the dilaton in the chiral representation. The $4_B + 4_F$ Yang Mills multiplet $W_\alpha{}^{(r)} \sim (a_m^{(r)} \mid \lambda_\alpha{}^{(r)} \mid\mid D^{(r)})$ with the index r belonging to the internal gauge group $G_{(r)}$ is defined as

$$W_{\alpha|}^{(r)} \;=\; -i\; \lambda_\alpha{}^{(r)}(x) \qquad \mathcal{D}_\beta W_{\alpha|}^{(r)} = -\varepsilon_{\beta\alpha} D^{(r)}(x) - i\; (\sigma^{mn}\varepsilon)_{\beta\alpha}\; f_{mn}{}^{(r)}(x). \tag{1.6}$$

The Yang-Mills prepotential V satisfies $W_\alpha = -\frac{1}{4}\, (\bar{\mathcal{D}}^2 - 8R)\; e^{-2V}\; \mathcal{D}_\alpha\; e^{2V}$. By constructing the chiral density $\mathcal{E} = e + ie\theta\sigma^a\bar\psi_a - e\theta^2(\bar M + \bar\psi_a\bar\sigma^{ab}\bar\psi_b)$ one finds the $12_B + 12_F$ minimal multiplet for the supergravity sector [18], which we denote by the supercurvature $R \sim (e_m{}^a \mid \psi_m{}^\alpha \mid\mid M,\; b_a)$, namely the graviton, the gravitino and two auxiliary fields [15]. The reducibel $8_B + 8_F$ linear multiplet

[1] We use the convention $\kappa^2 = 8\pi/M_{pl}^2 = 1$. Moreover we use the usual superspace notations $\int \equiv \int d^4\theta$ and $X_| \equiv X_{|\underline{\theta}=0}$ with $\int d^4\theta = -\frac{1}{4}\int d^2\theta(\bar{\mathcal{D}}^2 - 8R)$. We will refer to $\int d^4\theta$ as a D-density and to $\int d^2\theta$ as a F-density.

$$L \ \sim \ (C, b_{mn}, a_m^{(r)} \mid \varphi_\alpha, \lambda_\alpha^{(r)} \mid\mid D^{(r)}) \tag{1.7}$$

is the difference of the $4_B + 4_F$ Chern-Simons superfield $\Omega \sim (a_m^{(r)} \mid \lambda_\alpha^{(r)} \mid\mid D^{(r)})$ and the $4_B + 4_F$ real linear multiplet $l \sim (C, b_{mn} \mid \varphi_\alpha \mid\mid -)$. It satisfies the following two constraints:

$$(\bar{\mathcal{D}}^2 - 8R)\ L = -2\ k_{(r)} \ \ W^{\alpha(r)} W_{\alpha(r)} \qquad\qquad L = l - \ k_{(r)}\ \Omega^{(r)} = L^+ \tag{1.8}$$

The parameter $k_{(r)}$ denotes the normalization of the gauge group generators $tr\ T_{(r)} T_{(s)} = k_{(r)}\ \delta_{(r)(s)}$ and is in the context of string theory the level of the Kac-Moody current algebra of $G_{(r)}$. The linear mutliplet L is the $N = 1$ limit of the $N = 2$ vector-tensor multiplet [19]. Both multiplets have the same field content, but the $N = 2$ vector-tensor multiplet is irreducibel because of the extended supersymmetry. The reducibel parts of the linear superfield, namely the real linear superfield l and the Chern-Simons superfield Ω, satisfy $(\bar{\mathcal{D}}^2 - 8R)\ l = 0$ and $(\bar{\mathcal{D}}^2 - 8R)\ \Omega = 2 \ \mathrm{tr}\ W^\alpha W_\alpha$ respectively. This can be used to write a local F-density into a local D-density up to total derivatives: Consider the following Yang Mills action with an arbitrary chiral function $F(\Sigma)$

$$\int d^2\theta\ F(\Sigma)\ W^\alpha W_\alpha + \text{total derivatives} + h.c. = -2 \int d^4\theta \left\{ F(\Sigma) + \bar{F}(\bar{\Sigma}) \right\} \Omega \tag{1.9}$$

The RHS of (1.9) is *exactly* invariant under the following shift $F(\Sigma) \to F(\Sigma) \ + \ i\ \Theta$. On the LHS of (1.9) this holds only if one takes the boundary terms into account. The LHS of (1.9) *without* boundary terms contains at component level the CP-odd term $f\tilde{f}$ coupled to the axion. In perturbation theory these boundary terms can be ignored, but non-perturbatively this is not obvious. This has to be taken into account in the discussion of gaugino condensation. Furthermore we want to mention, that the real Yang-Mills Chern-Simons superfield Ω and the chiral Yang-Mills superfield W^α have the same field content. That is why the superfield representation of the component fields $(a_m^{(r)} \mid \lambda_\alpha^{(r)} \mid\mid D^{(r)})$ is not unique.
The linear multiplet L contains a real scalar C, which is called dilaton in this framework, its supersymmetric partner, the dilatino φ_α, a two-form antisymmetric tensor b_{mn} and the Yang-Mills Chern-Simons three-form $\omega_{3Y\,nml} = -\mathrm{tr}(a_{[l}\partial_m a_{n]} - \frac{2i}{3}\ a_{[l} a_m a_{n]})$:

$$\begin{aligned}
\ln L_| &= C(x) \\
\mathcal{D}_\alpha \ \ln L_| &= \varphi_\alpha(x) \\
\left[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\right] L_| &= -\frac{4}{3} e^C b_{\alpha\dot\alpha} + 4\ k_{(r)}\ \lambda_\alpha^{(r)} \bar\lambda_{\dot\alpha(r)} + \sigma_{k\alpha\dot\alpha} \Big\{ \varepsilon^{klmn} (\partial_n b_{ml} - \frac{1}{3}\ k_{(r)}\ \omega_{3Y\,nml}^{(r)} \\
&\quad + i\ e^C \psi_n \sigma_m \bar\psi_l) + 2i\ e^C (\psi_m \sigma^{mk} \varphi - \bar\psi_m \bar\sigma^{mk} \bar\varphi) \Big\} (x)
\end{aligned} \tag{1.10}$$

Note that L is invariant under the $U(1)_{PQ}$ symmetry. The duality transformed linear multiplet will be denoted as $S_R = S + \bar{S}$, where S is a chiral multiplet. We define

$$S_| \;=\; (e^{-C} \;+\; i\; a)(x) \qquad \mathcal{D}_\alpha S_| = \sqrt{2}\; \rho_\alpha(x) \qquad \mathcal{D}^2 S_| = -4\; f(x) \qquad (1.11)$$

In general the gauge group G has a product structure $G = \prod_{(r)} G_{(r)}$. Nevertheless for $G_{(r)}$ the gauge coupling is defined at tree level as $g_{(r)}^{-2} = k_{(r)}\; \langle e^{-C} \rangle$. So these physical parameters of the different gauge groups $G_{(r)}$ are related to each other at tree level [20,21]. The combination $S + \bar{S}$ is invariant under the $U(1)_{PQ}$ symmetry because it depends on the axion only due to derivative terms. On shell the supersymmetric duality of the dilaton transforms the $2_B + 2_F$ real linear multiplet $l \sim (C, b_{mn} \mid \varphi_\alpha \mid\mid -)$ to the $2_B + 2_F$ multiplet $S_R \sim (C, \partial_m a \mid \rho_\alpha \mid\mid -)$ without any difficulties. However, the off-shell structure of this duality relating two inequivalent off-shell theories to each other is still an open problem in many aspects, although the duality transformation can be performed off-shell.
All the multiplets we have introduced so far, except the linear multiplet, are irreducible. In the context of string compactifications it is interesting to build reducibel multiplets out of them and study their relationship to EQFT's with extended supersymmetry. We have already mentioned, that the linear multiplet is associated to the $N = 2$ vector-tensor multiplet. Furthermore the minimal multiplet and the real linear multiplet form the $16_B + 16_F$ multiplet $(R + l) \sim (e_m^{\ a}, C, b_{mn} \mid \psi_m^{\ \alpha}, \varphi_\alpha \mid\mid M,\; b_a)$, which was shown to be the $N = 1$ limit of a $N = 4$ EQFT [22]. In the end the minimal multiplet and the linear multiplet can combine to the $20_B + 20_F$ multiplet

$$(R + L) \;\sim\; (e_m^{\ a}, a_m^{\ (r)}, C, b_{mn} \mid \psi_m^{\ \alpha}, \lambda_\alpha^{\ (r)}, \varphi_\alpha \mid\mid M,\; b_a, D^{(r)}). \qquad (1.12)$$

We will couple the irreducible parts of this $20_B + 20_F$ multiplet to matter via the Kähler potential $K(\Sigma, \bar{\Sigma})$. The Kähler potential is as usual a real function depending on the supersymmetric matter multiplets Σ of the underlying EQFT.
In the $U_K(1)$-superspace formulation of $N = 1$ supergravity [2,16] the action contains three parts

$$\mathcal{L} \;=\; \mathcal{L}_{matter} + \mathcal{L}_{pot} + \mathcal{L}_{YM} \qquad (1.13)$$

with the three basic functions, namely the Kähler potential K, the superpotential ω and the gauge kinetic function $f_{(r)(s)}$:

$$\begin{aligned}
\mathcal{L}_{matter} &= m_i \int E[K_i] \\
\mathcal{L}_{pot} &= \frac{1}{2} \int \frac{E}{R}\; e^{K/2}\; \omega(\Sigma) + h.c. \\
\mathcal{L}_{YM} &= \frac{1}{2} \int \frac{E}{R}\; W^{(r)}\; f_{(r)(s)}\; W^{(s)} + h.c. \qquad (1.14)
\end{aligned}$$

The parameter m_i depends on the representation of the dilaton: It is useful in the following to introduce the parameter $n = 1/4$. Then m_i is given as $m_{linear} = 4n - 3$ and $m_{chiral} = -3$.
Moreover we will use the following variations in $U_K(1)$-superspace for a general superfield Z :

$$\delta_U Z = -\frac{\mathcal{W}_K(Z)}{2m}\,\frac{\partial K}{\partial U}\,\delta U\ Z, \tag{1.15}$$

The Kähler weights are given as $\mathcal{W}_K(E, L, l, \Omega, S, \Sigma, Y^3) = (-2, 2, 2, 2, 0, 0, 2)$, whereas the superfield $Y^3 = W^\alpha W_\alpha$ plays an important role in the context of gaugino condensation, because its lowest component is given by gaugino bilinears. In our superspace formulation the following identity holds: $\delta_{Y^3}\left(Y^3\ e^{-K/2}\right) = \delta Y^3 e^{-K/2}$. And by the use of (1.8) we find

$$\delta_{Y^3}\int d^4\theta\ E[K]\ f(\Omega) = -\frac{1}{2}\int d^2\theta\ E[K]\ \delta Y^3\left\{\frac{\partial f(\Omega)}{\partial\Omega} - \frac{\mathcal{W}_K(E)}{2m}\frac{\partial K}{\partial\Omega}f(\Omega)\right\}. \tag{1.16}$$

It turns out, that this is a very helpful identity in the discussion of gaugino condensation.

Before we will derive a general, duality-invariant scalar potential, we want to have a first look at the duality (1.1) for an ordinary bosonic QFT in the presence of Chern-Simons forms: We start with an unconstrained lagrangian

$$\mathcal{L}_u = \frac{1}{2}\,H^m H_m + \left(H^m - k_{(r)}\Omega^{(r)m}\right)\partial_m a \tag{1.17}$$

with $\Omega^{(r)m} = \frac{1}{3}\varepsilon^{mnpq}\omega_{3Y\,npq}$. Variation with respect to $a(x)$ yields $\partial_m(H^m - k_{(r)}\Omega^{(r)m}) = 0$ with the general solution $H^m = \varepsilon^{mnpq}\partial_n b_{pq} + k_{(r)}\Omega^{(r)m}$. This leads to the following action:

$$\mathcal{L}(b_{pq}, a) = \frac{1}{2}\,H^m H_m + \partial_m\left(a\ \varepsilon^{mnpq}\ \partial_n\ b_{pq}\right) \tag{1.18}$$

Note that (1.18) is an action of the antisymmetric tensor only, if one omits the boundary term. Furthermore $\mathcal{L}(b_{pq})$ is invariant under the $U(1)_{PQ}$ symmetry. Variation of $\mathcal{L}_u$ with respect to $H_m(x)$ yields $H_m = -\partial_m a$. This leads to a lagrangian, which contains a pseudoscalar instead of the antisymmetric tensor:

$$\mathcal{L}(\partial_m a) = -\frac{1}{2}\,\partial_m a\ \partial^m a - \ a\ k_{(r)}\ f^{(r)}_{mn}\ \tilde{f}^{mn}_{(r)} - \ \partial_m\left(a\ k_{(r)}\Omega^{(r)m}\right) \tag{1.19}$$

Here we used the usual definition $\tilde{f}_{mn}^{(r)} = \frac{1}{2}\varepsilon_{mnpq} f^{pq(r)}$ and the well-known identity $\partial_m \Omega^{(r)m} = -f^{(r)}_{mn}\tilde{f}^{mn}_{(r)}$. Again the resulting lagrangian is invariant under the $U(1)_{PQ}$ symmetry. If we consider a constrained lagrangian $\mathcal{L}_c$, which is $\mathcal{L}_u$ satisfying the on-shell constraint: $H^m = \varepsilon^{mnpq}\, \partial_n\, b_{pq} \; + \; k_{(r)}\Omega^{(r)m} = - \, \partial^m a$, then we can relate the two models to each other directly - one containing an antisymmetric tensor and the other a pseudoscalar.
So far we have only summarized well-known results that are useful in the following. We want to discuss the duality now in the framework of $N = 1$ superspace: The tree-level Kähler potential under discussion in the linear representation of the dilaton will be $\tilde{K} = 4n\,\ln(L/2) + K(\Sigma, \bar{\Sigma})$. It already includes the tree level gauge kinetic function, but therefore it is not a well-defined Kähler potential in the sense, that it does not fulfill the Kähler condition [13]. Furthermore at one loop the Wilsonian gauge coupling function is determined by a holomorphic function of the chiral fields Σ: $f^{[1]}_{(r)(s)} = \delta_{(r)(s)}\; f^{[1]}(\Sigma)$. We assume here, that in perturbation theory the Wilsonian gauge coupling function is beyond tree level independent of the dilaton [26]. The lagrangian considered here is evaluated at component level in [2]. The part of the lagrangian containing the auxiliary fields reads

$$\mathcal{L}_{aux}/e = \frac{4n-3}{9} M\bar{M} + G_{i\bar{j}} F^i \bar{F}^{\bar{j}} + e^{\Gamma/2}\left(\frac{4n-3}{3}(M+\bar{M}) + F^i G_i + \bar{F}^{\bar{j}} \bar{G}_{\bar{j}}\right), \quad (1.20)$$

where G is the well-defined dual G-function: $G = K(\Sigma, \bar{\Sigma}) + \ln|\omega|^2$. So the scalar potential from this part after elimination of the auxiliary fields is

$$V_1 \;\; = \;\; e^{\Gamma}\; (G_i\; G^{i\bar{j}} G_{\bar{j}} + 4n - 3) \quad (1.21)$$

The second part of the potential is the sum of all monomials coupling the dilaton C only to tr λ^2:

$$V_2 \;\; = \;\; (n\; \mathrm{tr}\lambda^2 - e^{\Gamma/2}\Gamma_L)\; \Gamma^{LL}\; (n\; \mathrm{tr}\bar{\lambda}^2 - e^{\Gamma/2}\Gamma_L) - 4n\; e^{\Gamma} \quad (1.22)$$

Derivatives with respect to the linear multiplet are defined as $\partial_L = \frac{\partial}{\partial(2/L)}$. The whole potential can be rewritten as

$$V \;\; = \;\; \left(n\; \delta_{iL}\; tr\; \lambda^{\alpha}\lambda_{\alpha} - e^{\Gamma/2}\Gamma_i\right)\; \Gamma^{i\bar{j}}\left(n\; \delta_{\bar{j}L}\; tr\; \bar{\lambda}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - e^{\Gamma/2}\Gamma_{\bar{j}}\right) - 3e^{\Gamma}. \quad (1.23)$$

The indices $i, \bar{j}$ include derivatives with respect to the linear muliplet now. And δ_{iL} is the Kronecker-delta. The result reduces to the known potential in the chiral limit $n = 0$.
Performing now the duality transformation our lagrangian changes to an unconstrained lagrangian $\mathcal{L}_u$ by changing l to U, where U is unconstrained, and by adding a lagrange multiplier $\mathcal{L}_{lm}$. The lagrange multiplier contains the unconstrained field U and the $S + \bar{S}$ multiplet.

$$\mathcal{L}_u \;=\; m \int E \,\left(1 - \frac{2n}{m}\, U\, (S+\bar{S})\right) + \mathcal{L}_{pot} \tag{1.24}$$

Variation with respect to $S+\bar{S}$ yields the old theory in the linear representation of the dilaton. Variation with respect to U yields the chiral representation of the dilaton. Note that the variations depend on the representation of the dilaton. In the $U_K(1)$-superspace the torsion constraints and consequently the solution of the Bianchi-identities depend on the representation of the dilaton: In the notation of [16] the superdeterminant of the vielbein E, for instance, gets rescaled as $E' = E\,(X\bar{X})^2$ with $X = \bar{X} = e^{K/4m}$. The rescalings are chosen in such a way, that the whole theory is automatically Einstein normalized. [2] Thus, we find with $\tilde{K}(U) = 4n \ln(U - k_{(r)}\Omega^{(r)})$ the *duality relation* $S+\bar{S} = 2/L$. Inserting this relation one ends up with the action in the chiral representation of the dilaton:

$$\begin{aligned} \mathcal{L} \;&=\; -3 \int E[K] \left(1 + \frac{2n}{3} k_{(r)}\Omega^{(r)}\, (S+\bar{S})\right) \\ &\quad + \left\{ \frac{1}{2}\int \frac{E}{R}\, e^{K/2}\, \omega(\Sigma) + \frac{1}{2}\int \frac{E}{R}\, W^{(r)}\, f^{[1]}_{(r)(s)}\, W^{(s)} \;+ h.c. \right\} \end{aligned} \tag{1.25}$$

This action is manifestly invariant under the $U(1)_{PQ}$ symmetry of the dilaton in the chiral representation. It depends only on $\partial_m a(x)$, because the theory in the L-representation only knows about the field strength of the antisymmetric tensor. One open problem is encoded in the fact, that off-shell we have in the S-representation the corresponding auxiliary field f, which is absent in the linear representation. But since we are discussing the supersymmetric duality only on-shell, we have to eliminate the auxiliary fields via their equations of motion. It is quite interesting, that the remaining scalar potential is the same than the one in the L-picture as we will show now: The Kähler potential and the gauge coupling function are given in the S-representation by $K = -4n\, \ln(S+\bar{S}) + K(\Sigma,\bar{\Sigma})$ and $f_{(r)(s)} = n\ \delta_{(r)(s)}\ \left(f^{[0]} + f^{[1]}\right)$ respectively. The tree-level gauge coupling function $f^{[0]} = (S+\bar{S})\ k_{(r)} = 2\ k_{(r)}/L$ is part of a D-density. This point of view was already very successful in the discussion of non-holomorphic field-dependent contributions to the gauge coupling function [5]. After performing the usual Kähler transformation to go to the G-function the auxiliary part of the lagrangian is given by

$$\begin{aligned} \mathcal{L}_{aux}/e \;&=\; -\frac{1}{3}M\bar{M} + G_{i\bar{j}}F^i\bar{F}^{\bar{j}} - n\ F^i\ \delta_{iS}\ tr\ \lambda^\alpha\lambda_\alpha - n\ \bar{F}^{\bar{j}}\ \delta_{\bar{j}\bar{S}}\ tr\ \bar{\lambda}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} \\ &\quad + e^{G/2}\left(F^iG_i + \bar{F}^{\bar{j}}\bar{G}_{\bar{j}} - M - \bar{M}\right), \end{aligned} \tag{1.26}$$

with $F^i\ \delta_{iS} = f$. Eliminating the auxiliary fields via their equation of motion leads to the scalar potential:

[2] In Lorentz-superspace the Weyl-rescaling of the graviton depends on the representation of the dilaton [13].

$$V = \left(n\ \delta_{iS}\ tr\ \lambda^\alpha\lambda_\alpha - e^{G/2}G_i\right)\ G^{i\bar{j}}\left(n\ \delta_{\bar{j}\bar{S}}\ tr\ \bar{\lambda}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - e^{G/2}G_{\bar{j}}\right) - 3e^G \quad (1.27)$$

This potential includes the usual scalar potential of ordinary matter fields of [24] and is precisely (1.23) - the scalar potential derived in the linear representation of the dilaton.

Now we can use the duality-invariant scalar potential to study gaugino condensation in the two superfield-representations of the dilaton. We will restrict ourselves first of all to the case of one gauge group and take $k_{(r)} = 1$ for simplicity. We start in the linear representation of the dilaton: The superpotential consists of two parts: The first part is the so called quantum part and has its origin in chiral and conformal anomalies [25]. The second part represents the one-loop threshold corrections to the gauge coupling function. Note that the superpotential is explicitly dilaton-free, because it is defined to be a chiral function. However it depends implicitly on the dilaton through the Kähler potential, which can enter the superpotential[3]. The two parts combine to the following effective superpotential

$$\omega(\Sigma) = \frac{1}{\beta}\ Y^3 e^{-\tilde{K}/2}\ \ln\left\{c^6\ e^{\beta n f^{[1]}}\ Y^3\ e^{-\tilde{K}/2}\right\} \quad (1.28)$$

with $\beta = -24\pi^2/b\ n$, where b denotes the $N = 1$ β-function coefficient. By the use of (1.16) we find the equations of motion for Y^3. Because the action contains the Yang-Mills Chern-Simons superfield Ω and its chiral projection Y^3 at the same time, the equation of motion splits into a non-holomorphic and a holomorphic part. This property can be used to introduce non-holomorphic terms into the superpotential. After an appropriate Kähler transformation with $\tilde{K} \to \Gamma = \tilde{K} + \ln|\omega|^2$ we have

$$\lambda^\alpha\lambda_\alpha = e^{\Gamma/2}\ \frac{1}{1/\beta + n f^{[0]}}. \quad (1.29)$$

So we find immediately, that in the weak coupling limit $f^{[0]} \to \infty$ the vacuum expectation value of the gaugino bilinears vanishes and consequently gaugino condensation does not take place. The scalar potential is given by (1.23) and by the use of the equations of motion for the gaugino bilinears we can integrate them out. At this point we want to show, that the results given here are invariant under the duality transformation: Performing the duality transformation the action is given by (1.25). Again the tree-level gauge coupling function is part of a D-density. Analogous to the linear representation of the dilaton the effective superpotential is given by (1.28). Using (1.16) we find the equation of motion for Y^3 in the S-representation. This yields again (1.29) with G instead of Γ, of course. Again it is now straightforward to integrate out the

[3]This is also the case in the superconformal approach, where the compensators are functions of the Kähler potential.

gaugino bilinears using (1.27). In the end the procedure of integrating out the gaugino bilinears via their equations of motion is not affected by the duality transformation.

As a concrete example we turn now to the discussion of orbifold models describing the compactification of the heterotic string from ten dimensions down to four [27]. We will focus on (2,2) symmetric Z_N orbifold compactifications without Wilson lines. In these models occurs the generic gauge group $E_8 \otimes E_6 \otimes H$ with $H = \{SU(3), SU(2) \times U(1), U(1)^2\}$. Orbifold compactifications posses various continuous parameters, called moduli, corresponding to marginal deformations of the underlying conformal field theory. These moduli enter the EQFT and they take their values in a manifold $\mathcal{M}$ called moduli space. For models with $N = 1$ space-time supersymmetry $\mathcal{M}$ is, locally, a Kählerian manifold. Thus, the corresponding Kähler potential describes the coupling of the moduli to $N = 1$ supergravity in the EQFT under consideration. The EQFT must respect target space modular symmetries (for a review see [23]) induced by the target space duality group. For orbifold compactifications the target space duality group is often given by the modular group $PSL(2, \mathbf{Z})$, acting on one chiral field T as

$$T' \;=\; \frac{a\,T - i\,b}{i\,c\,T + d} \qquad ad - bc = 1 \qquad a, b, c, d \in \mathbf{Z} \tag{1.30}$$

where T corresponds to an internal, overall modulus: $T = R^2 + iB$. For simplicity we will discuss (2,2) symmetric Z_N orbifolds without (1,2) moduli. At the massless level of these orbifolds one finds (1,1) moduli and matter fields. These fields can be in general (un-) twisted and (un-) charged. In the following the three diagonal elements of the untwisted uncharged (1,1) moduli are denoted by T^A, whereas all other charged (uncharged) fields are given by Q^I_{ch} (Q^I_{uch}). We will only be interested in the lowest components of the superfields in order to study the vacuum structure of our theory. Since in the Wess-Zumino gauge we have $V_{|WZ} = 0$, it is not necessary for us to investigate the role of the charged and uncharged matter separately. So we define $Q^I = \{Q^I_{ch}, Q^I_{uch}\}$ and use this definition in an obvious way.
Now we must specify the Kähler potential $\tilde{K}$ and the effective superpotential ω. The Kähler potential $\tilde{K}$ can be generically expanded in powers of Q^I:

$$\tilde{K} \;=\; \ln(L/2) \;+\; \hat{K}(T) \;+\; Z_{I\bar{J}}(T,\bar{T})\, \bar{Q}^{\bar{J}}\, e^{2V}\, Q^I \;+\; \mathcal{O}\left((\bar{Q}Q)^2\right) \tag{1.31}$$

We choose the matter metric in the following diagonal form

$$Z_{I\bar{J}}(T,\bar{T}) = \delta_{I\bar{J}}\; Z^I(T,\bar{T}) \qquad\qquad Z_I(T,\bar{T}) = \prod_A\, (T^A + \bar{T}^A)^{-{q_I}^A}, \tag{1.32}$$

whereas the metric for the moduli is given by $\hat{K}_{i\bar{j}}$ with $\hat{K} = -\sum_{A=1}^{3}\, \ln(T^A + \bar{T}^A)$. We will use the usual notation $K^{i\bar{j}} = K^{-1}_{i\bar{j}}$ also for the matter metric. The parameters ${q_I}^A$ are the so called modular weights [28]. The superpotential has the following structure

$$\omega(\Sigma) \;=\; P(Q,T) \;+\; \sum_{(r),A} \omega^{A}_{(r)}(\Sigma). \tag{1.33}$$

More precisely we have superpotentials of the following form in mind

$$P(Q,T) \;=\; \frac{1}{3}\, Y_{IJK}(T)\; Q^I\; Q^J\; Q^K \tag{1.34}$$

$$\omega^{A}_{(r)}(\Sigma) \;=\; \frac{1}{\beta^{A}_{(r)}}\; Y^3 e^{-\tilde{K}/2}\, \ln\left\{ c^{A6}_{(r)}\; Y^3\; e^{-\tilde{K}/2}\; e^{n\beta^{A}_{(r)} f^{[1]A}_{(r)}} \right\}, \tag{1.35}$$

with the definition $\beta^{A}_{(r)} = -24\pi^2/b^{A}_{(r)} n$. The canonical dimensions of the fields are $dim(T^A, l, \Omega, Y, Q^I) = (0,0,2,1,1)$ and the transformation properties under the target space duality group (1.30) with $F_A = \ln(icT_A + d)$ read

$$T^A + \bar{T}^{\bar{A}} \;\longrightarrow\; (T^A + \bar{T}^{\bar{A}})\; e^{-(F_A + \bar{F}_A)} \qquad \text{(no sum over } A\text{)}$$

$$Z_{I\bar{J}}(T,\bar{T}) \;\longrightarrow\; Z_{I\bar{J}}(T,\bar{T})\; e^{{q_I}^A (F_A+\bar{F}_A)} \qquad \text{(sum over } A\text{)}$$

$$Q_I \;\longrightarrow\; Q_I\; e^{-{q_I}^A F_A} \qquad \text{(sum over } A\text{)} \tag{1.36}$$

Therefore the target space duality transformations act just as Kähler transformations. The potential is given in general by (1.23). Note that this is a 'closed' formula without specifying the superpotential:

$$\begin{aligned} G_i\, G^{i\bar{j}} G_{\bar{j}} \;=\; & \frac{1}{t}\, \left\{ \frac{{T_R}^{A2}}{t}\, |t\, \partial_{T^A} \ln \omega \;-\; \frac{1}{{T_R}^{A}}|^2 \;-\; \frac{1}{t}\; |{q_I}^{A2}\; Q^2|^2 \right. \\ & + Z^{I\bar{J}}\; \Big({q_I}^{A2}\; Q^2\; |\partial_{Q^I} \ln \omega|^2_{I\bar{J}} \;+\; t\; |Z_{I\bar{K}} \bar{Q}^{\bar{K}} \;+\; \partial_{Q^I}\, \ln \omega|^2_{I\bar{J}} \\ & - |{q_I}^{A} Z_{I\bar{K}} \bar{Q}^{\bar{K}} \;+\; \partial_{Q^I}\, \ln \omega|^2_{I\bar{J}} \\ & \left. + |{q_I}^{A} Z_{I\bar{K}} \bar{Q}^{\bar{K}}\; {T_R}^{A} \partial_{T^A} \ln \omega \;+\; \partial_{Q^I} \ln \omega|^2_{I\bar{J}} \Big) \right\} \end{aligned} \tag{1.37}$$

All indices are contracted and the following definitions have been used: $Q^2 = Z_{I\bar{J}}(T,\bar{T}) \bar{Q}^{\bar{J}} Q^I$ and $t = 1 - {q_I}^{A2}\; Q^2$. The equation of motion for the gaugino bilinears yields now

$$tr\; \lambda^\alpha \lambda_\alpha \;=\; \frac{1}{3}\; e^{\Gamma/2}\; tr\; \sum_{A=1}^{3} \left(\frac{1}{\beta^{A}_{(r)}} \;+\; n\; f^{[0]A}_{(r)} \right)^{-1} \tag{1.38}$$

with $f^{[0]A}_{(r)} = f^{[0]}_{(r)}/3 = 2k_{(r)}/3L = k_{(r)} S_R/3$. In the weak coupling limit gaugino condensation still disappears. Using the equation of motion for the gaugino bilinears leads to non-holomorphic contributions to the superpotential:

$$\omega^A_{(r)} \;=\; -e^{-1}\; {c^A_{(r)}}^{-6}\; e^{-n\beta^A_{(r)}(f^{[0]A}_{(r)}+f^{[1]A}_{(r)})} \left(\frac{1}{\beta^A_{(r)}} \;+\; n\; f^{[0]A}_{(r)} \right) \tag{1.39}$$

It is important to stress, that $f^{[0]A}_{(r)}$ can be expressed by the use of the equations of motion for Y^3 in a pure holomorphic way. This property is directly related to the fact, that we have to deal with the non-holomorphic Chern-Simons superfield Ω and its holomorphic projection Y^3 at the same time. As a consequence the remaining scalar potential has the desired non-perturbative structure $V \sim e^{-(S+\bar{S})} = e^{-2/L} \sim e^{-1/g^2_{tree}}$, if the theory is asymptotical free ($\beta^A_{(r)} > 0$). Our approach produces this functional dependence on the dilaton as an overall factor in (1.39). So the $U(1)_{PQ}$ symmetry is still unbroken. The matter fields we have introduced are quantum fields with vanishing vacuum expectation value. Because we are interested in the vacuum structure of our theory, we take the limit $Q^I \to 0$.
Up to now it was not necessary to specify the one-loop contribution to the gauge-coupling function. Following [26] we have

$$f^{[1]A}_{(r)}(\Sigma) \;=\; -\,\frac{b^A_{(r)}}{8\pi^2}\,\ln \eta^2(T^A), \tag{1.40}$$

where $\eta(T^A)$ is the well-known Dedekind function and reflects the one-loop threshold contributions of momentum and winding states of the underlying string theory. For simplicity we have assumed, that the coefficient which appears in the threshold correction is the $N = 1$ β-function coefficient. This is the case for a hidden gauge group of pure Yang-Mills theory like E_8 in the absence of Green-Schwarz terms. The inclusion of a Green-Schwarz term was discussed in the last reference of [10].
Using (1.38) the scalar potential is given as

$$V(C,T^A) = \frac{e^C}{2}\prod_{A=1}^{3}({T_R}^A)^{-1}|\omega|^2 \left\{ \sum_{A=1}^{3} {T_R}^{A2}\; |\frac{3\hat{G}_2(T^A)}{2\pi} + \frac{2}{{T_R}^A}|^2 + k(C) - 3 \right\} \tag{1.41}$$

with the Eisenstein function $\hat{G}_2(T) = G_2(T) \;-\; 2\pi/T_R$ and

$$k(C) \;=\; |\; tr\; \sum_{A=1}^{3} \frac{\beta^A_{(r)}}{6e^C \;+\; \beta^A_{(r)}\; k_{(r)}} + 1\;|^2. \tag{1.42}$$

We have calculated the effective scalar potential for factorizable Kählerian moduli spaces $\mathcal{M}$ of the form $\mathcal{M} = \mathcal{M}^{(1,1)}_{dilaton} \;\otimes\; \mathcal{M}'$ with $\mathcal{M}^{(1,1)} = SU(1,1)/U(1)$. It is well-known that supersymmetry can be broken via effective scalar potentials of the form (1.41) with (1.40) [8]. This property does not depend on the representation of the dilaton [13].

To conclude, we have shown in this paper, that there exists a consistent on-mass-shell formulation of gaugino condensation in local $N = 1$ string effective field theories in four dimensions.
We studied the one-loop anomalous contribution to the Wilsonian gauge coupling, discussed by Dixon, Louis and Kaplunovsky [26], and the anomalous contribution to the effective action of Taylor, Veneziano and Yankielowicz [25]. Both can combine to an effective superpotential. Using this effective superpotential we studied supersymmetry breaking via gaugino condensation in (2,2) symmetric Z_N orbifolds. In our approach we have integrated out the gaugino bilinears via their equations of motion. By the use of the Yang-Mills Chern-Simons superfield and its chiral projection the equations of motion split into a holomorphic and a non-holomorphic part. This result, which clearly only holds on-shell, leads to the non-perturbative structure of the effective scalar potential with the typical $e^{-1/g^2_{tree}}$ behaviour. Moreover we have shown that our approach is independent of the superfield-representation of the dilaton and preserves the $U(1)_{PQ}$ symmetry.
Finally, we want to mention that the duality invariant off-shell formulation of gaugino condensation is still an open problem in many aspects.

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